35.18 An energy efficiency initiative being evaluated for a 2,500 ton chiller plant running 24/7 is expected to improve the total average annualized COP from 4 to 5. The project budget is $800,000. Capital can be borrowed at an interest rate of 5%. Electricity costs are estimated at $0.14 per kWH. What is the loan duration that should be used if the initiative is required to be cash flow neutral?

A. 17 months

B. 18 months

C. 19 months

D. 20 months

The electricity to run the chiller plant consists of the power to run the compressors, which is $\dot{W}_{in}$ in the Coefficient of Performance (COP) formula. Rearrange the COP and solve the input power before and after the initiative. Arbitrarily call these Option 1 and Option 2. Convert units to KW.

$$COP_R = \frac{\dot{Q}_{in}}{\dot{W}_{in}} \rightarrow \dot{W}_{in} = \frac{\dot{Q}_{in}}{COP_R}$$

$$\dot{W}_{in,1} = \frac{(2500tons)\left(12,000\frac{Btu}{hr \cdot ton}\right)}{(4)\left(3412\frac{Btu}{hr \cdot KW}\right)} = 2198KW$$

$$\dot{W}_{in,2} = \frac{(2500tons)\left(12,000\frac{Btu}{hr \cdot ton}\right)}{(5)\left(3412\frac{Btu}{hr \cdot KW}\right)} = 1758KW$$

Calculate the annual cost savings associated with the reduction in demand. Multiply by time and the rate of electricity.

$$Annual\,Cost\,Savings = (2198KW - 1758KW)\,(8760hr)\left(\frac{\$0.14}{KW \cdot hr}\right) = \$539,600$$

Look up Economic Factor Conversions and find the option for Capital Recovery which converts to A given P, where A is an annualized payment and P is a lump sum of principal borrowed. The value of the conversion factor is a function of the interest rate and term length and is written as $(A/P, i\%, n)$. Since A and P are known, solve for the value of the factor.

$$A = (A/P, 5\%, n)\,P$$

$$(A/P, 5\%, n) = \frac{A}{P} = \frac{\$539,600}{\$800,000} = .6739$$

Set the value of the factor equal to the formula used for calculating it when both interest rate and term are known. In this case, the interest rate is known and the term length needs to be determined.

$$(A/P, 5\%, n) = \frac{i\,(1+i)^n}{(1+i)^n - 1} = .6739$$

Substitute $i = .05$:

$$\frac{(.05)\,(1.05)^n}{(1.05)^n - 1} = .6739$$

Rather than tackle the challenging algebra and potentially wasting time (see alternate solution below), consider an iterative trial and error approach where some answers may be quickly eliminated. As a starting point, determine the simple payback for the initiative:

$$Simple\,Payback = \frac{\$800,000}{\$539,600} = 1.48years\left(\frac{12months}{year}\right) \approx 18months$$

The simple payback does not account for any interest paid over time, therefore it is a more favorable representation of the financials from the perspective of the borrower. Answer choice A, $n = 17months$, can be eliminated on the basis that the term cannot be more favorable than the simple payback would imply.

It is difficult to eliminate any other choices; try all remaining answer choices to see which term length comes closest to the calculated value for $(A/P, 5\%, n)$. The units for the term length should be years to align with the interest rate, which is annual.

$$n = 18months\left(\frac{1year}{12months}\right) = 1.5years$$

$$\frac{(.05)\,(1.05)^{1.5}}{(1.05)^{1.5} - 1} = .7085$$

$$n = 19months\left(\frac{1year}{12months}\right) = 1.58years$$

$$\frac{(.05)\,(1.05)^{1.58}}{(1.05)^{1.58} - 1} = .6726$$

Since $.6726 < .6739 < .7085$, there is no need to try $n = 20months$.
$.6739$ is closer to $.6726$, so choose $n = 19months$.

Alternate Solution:

$$\frac{(.05)(1.05)^n}{(1.05)^n - 1} = .6739$$

Divide by .05:

$$\frac{(1.05)^n}{(1.05)^n - 1} = 13.478$$

Take the inverse of both sides:

$$\frac{(1.05)^n - 1}{(1.05)^n} = .0742$$

Break left side into two terms:

$$1 - \frac{1}{(1.05)^n} = .0742$$

Simplify:

$$\frac{1}{(1.05)^n} = .9258$$

Take the inverse of both sides:

$$(1.05)^n = 1.08$$

Recall from algebra that solving for an exponent involves a logarithm. Write an equivalent relationship to the above using logs:

$$n = log_{1.05}1.08$$

In order to solve a log with a base other than 10 or e, rewrite as a ratio of two logs with base 10:

$$n = \frac{log_{10}1.08}{log_{10}1.05} = 1.58years$$

Convert to months:

$$n = 1.58years\left(\frac{12months}{1year}\right) = 19months$$

Answer C